

- PHILIPP HIERONYMI, *Dichotomies for expansions of the real field*.
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Let $\overline{\mathbb{R}} = (\mathbb{R}, +, \cdot)$ be the field of real numbers. In this talk, the following characterization of expansions of $\overline{\mathbb{R}}$ by closed and discrete sets will be presented.

THEOREM 1 ([2]). *Let \mathcal{R} be an o-minimal expansion of $\overline{\mathbb{R}}$ and let $D \subseteq \mathbb{R}$ be closed and discrete. Then either*

- (\mathcal{R}, D) defines \mathbb{Z} or
- every subset of \mathbb{R} definable in (\mathcal{R}, D) has interior or is nowhere dense.

We will discuss possible generalizations of this theorem. As an application, we consider the following extension of the results from [1].

THEOREM 2 ([3]). *Let Γ be a finitely generated subgroup of \mathbb{C}^\times . Then either*

- $(\overline{\mathbb{R}}, \Gamma)$ defines \mathbb{Z} or
- there is $a \in \mathbb{R}^{>0}$ such that every open subset of \mathbb{R} definable in $(\overline{\mathbb{R}}, \Gamma)$ is already definable in $(\overline{\mathbb{R}}, a^{\mathbb{Z}})$.

[1] OLEG BELEGRADEK, BORIS ZILBER, *The model theory of the field of reals with a subgroup of the unit circle*, **Journal of the London Mathematical Society**, (2) 78 (2008) 563-579

[2] PHILIPP HIERONYMI, *Defining the set of integers in expansions of the real field by a closed discrete set*, **Proceedings of the American Mathematical Society**, 138 (2010) 2163-2168

[3] PHILIPP HIERONYMI, *Expansions of the real field by finitely generated subgroups of the complex field*, **in preparation**,