▸ JULIA F. KNIGHT, *Comparing classification problems.*
University of Notre Dame.
E-mail: knight.1@nd.edu.

A great deal of work in mathematics involves trying to classify the objects in some class, up to some important equivalence relation. We focus on classes of countable structures of familiar kinds, with isomorphism as the equivalence relation. We consider concrete ways to compare the classification problems. Different branches of logic have provided different approaches. One model theoretic approach involves cardinality—we compare the number of isomorphism types. Thus, $\mathbb{Q}$-vector spaces are easier to classify than undirected graphs.

Friedman and Stanley [5] developed an approach that involves Borel embeddings and "Borel cardinality". This approach tells us that classification for fields of characteristic 0 and finite transcendence degree, and for Abelian p-groups, is easier than for graphs, or for linear orderings, even though each of these classes has $2^{\aleph_0}$ isomorphism types of countable members. Borel cardinality does not distinguish among classes with $\aleph_0$ isomorphism types.

In [1], there is another approach, involving Turing computable embeddings and "effective cardinality". This approach tells us that number fields are easier to classify than $\mathbb{Q}$-vector spaces. The results in [6] suggest a uniform way to look for non-embeddability results, considering the form of the sentences that distinguish among non-isomorphic members of the different. In [2], we re-work in this way the result of Friedman and Stanley on non-embeddability of graphs in Abelian p-groups. We say that a class K has "Ulm type" if non-isomorphic members of K can be distinguished by infinitary sentences lying in the thinnest admissible set that contains the ordinals computable from the structures.

In [4], S-D. Friedman, Fokina, and Törnquist considered Σ^1_1 equivalence relations on ω. Among these are the isomorphism problems for *computable* members of various familiar classes, where the structures are identified with their indices. We may use Turing computable embeddings and effective cardinality to compare equivalence relations. Some distinctions are lost in this setting. In [3], it is shown that Abelian p-groups now lie on top—the effective cardinality is the same as for graphs.

[1] W. Calvert, D. Cummins, J. F. Knight, and S. Miller, "Comparing classes of finite structures", *Algebra and Logic*, vol. 43(2004), pp. 365-373.

[2] E. Fokina, J. F. Knight, C. Maher, A. Melnikov, and S. M. Quinn, "Classes of Ulm type, and relations between the class of rank-homogeneous trees and other classes", pre-print.

[3] E. B. Fokina, S-D. Friedman, V. Harizanov, J. F. Knight, C. McCoy, A. Montalbán, "Isomorphism and bi-bmbeddability relations on computable structures", pre-print.

[4] E. B. Fokina, S-D. Friedman, and A. Törnquist, "The effective theory of Borel equivalence relations", *Annals of Pure and Appl. Logic*, vol. 161(2010), pp. 837-850.

[5] H. Friedman and L. Stanley, "A Borel reducibility theory for classes of countable structures", *J. Symb. Logic*, vol. 54(1989), pp. 894-914.

[6] J. F. Knight, S. Miller (Quinn), and M. Vanden Boom, "Turing computable embeddings", *J. Symb. Logic*, vol. 73(2007), pp. 901-918.