▶ JAN SEBESTIK,

The significance of Bolzano's Beyträge zu einer begründeteren Darstellung der Mathematik.

CNRS Paris.
E-mail: sebestik@flu.cas.cz.

Two of the first Bolzano's publications have permanent interest: the Rein analytischer Beweis (1817) which inaugurates the arithmetization of analysis, and the Beyträge zu einer begründeteren Darstellung der Mathematik (1810). While the first one was noticed by Weierstrass, the second one, going against the spirit of the dominant Kantian and postkantian philosophy, was completely neglected.

Written in the Leibnizian spirit, the Beyträge develop a new philosophy of mathematics and should influence the philosophy of the XIXthe century in a similar way as Wittgenstein's Tractatus did in the XXth century. His main innovations consist in a new definition of mathematics, the claim of the objectivity of mathematics, the first modern study of an axiomatic system, an innovative theory of definition, a proof theory whose core is the the idea of grounding (Begründung), the criticism of Kant's philosophy of mathematics and, in the second unfinished part, published only in 1975, the first outline of his doctrine of collections, worked out later in the Grössenlehre. In the Beyträge, Bolzano sets up not only some of his fundamental concepts, but the structure of his work that will be expanded in the Wissenschaftslehre and in ber die Mathematische Lehrart.

The, unfinished second Lieferung of Bolzano's Beyträge, published only in 1975, brings an outline of Bolzano's theory of collections. Under the title Universal mathematics, Bolzano tries to determine the laws governing all things. It is in fact a problem of ontology and Bolzano starts by examining the ontological principles established in the Leibnizian-Wolffian school and in Kant. According to Bolzano, the validity of all these principles is limited and therefore he sets up his own principles: 1) the principle of thinking-together (Zusammendenken) according to which any thing can be thought together with any other thing, 2) the principle of relation stating that any thing is in a certain relation with any other thing. The first principle reminds the famous Cantor's definition of a set. Bolzano's concept of collection thus becomes the fundamental concept of mathematics. His doctrine will be worked out in the Grssenlehre in the 1830ties and in the Paradoxes of the infinite (1851) which will be one of the most important sources of Cantor's and Dedekind's set theories.