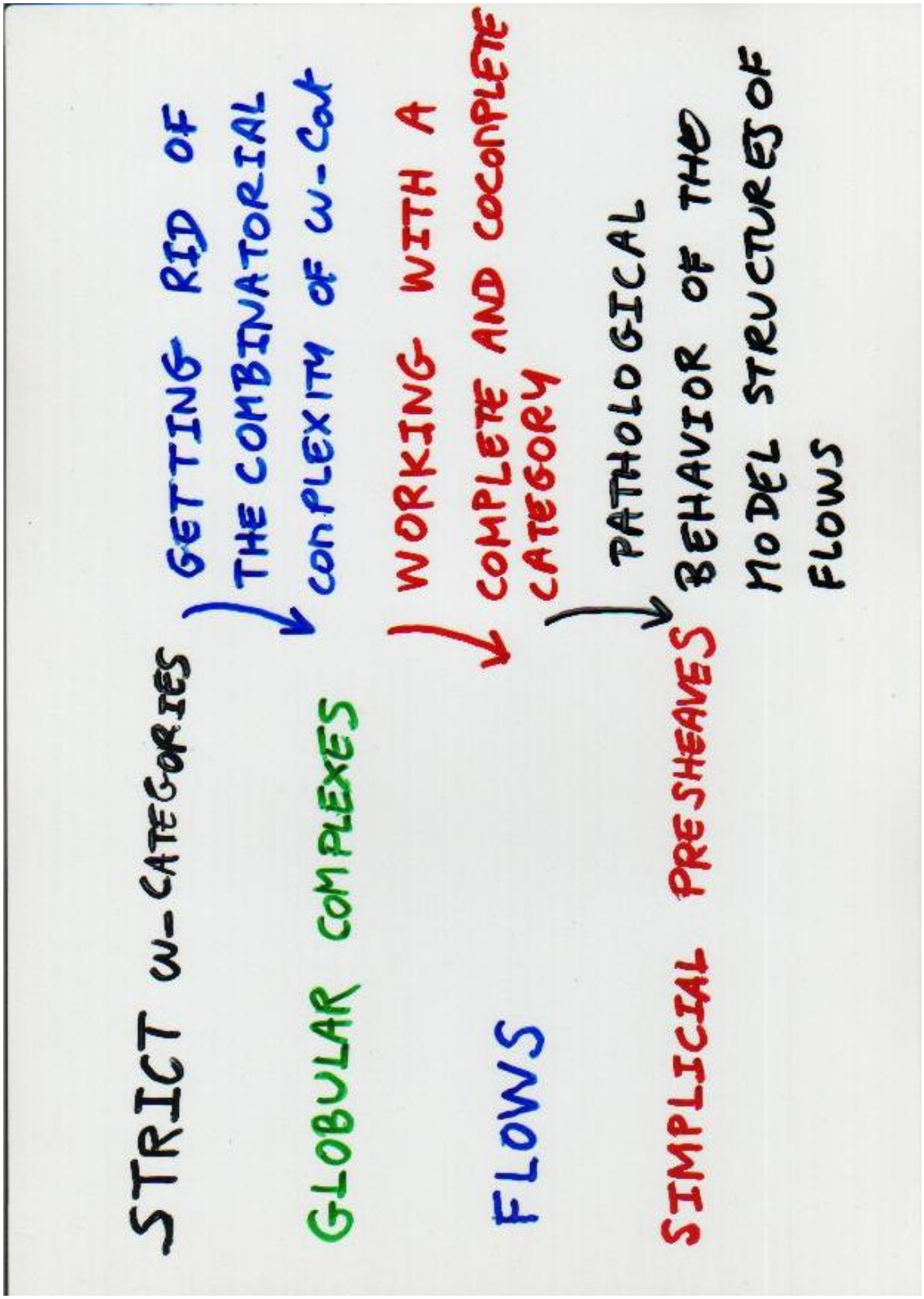
FLOW DOES NOT MODEL FLOWS UP TO WEAK DIHOMOTOPY (9 TRANSPARENCIES) TALK GIVEN THE 4TH OF MAY 2004, PARIS

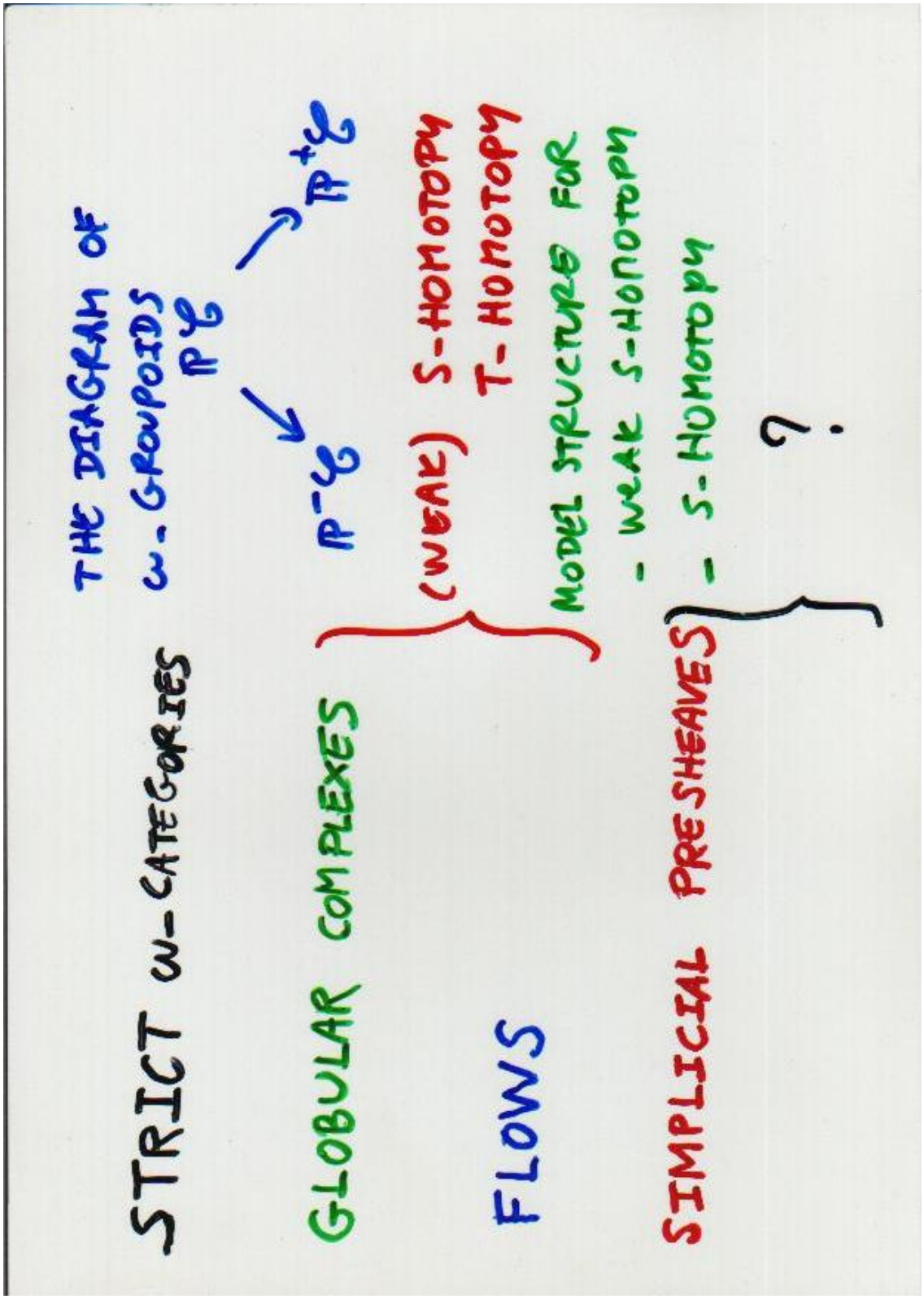
PHILIPPE GAUCHER

Abstract. We prove that the category of flows cannot be the underlying category of a model category whose corresponding homotopy types are the flows up to weak dihomotopy. Some hints are given to overcome this problem. In particular, a new approach of dihomotopy involving simplicial presheaves over an appropriate small category is proposed. This small category is obtained by taking a full subcategory of a locally presentable version of the category of flows.

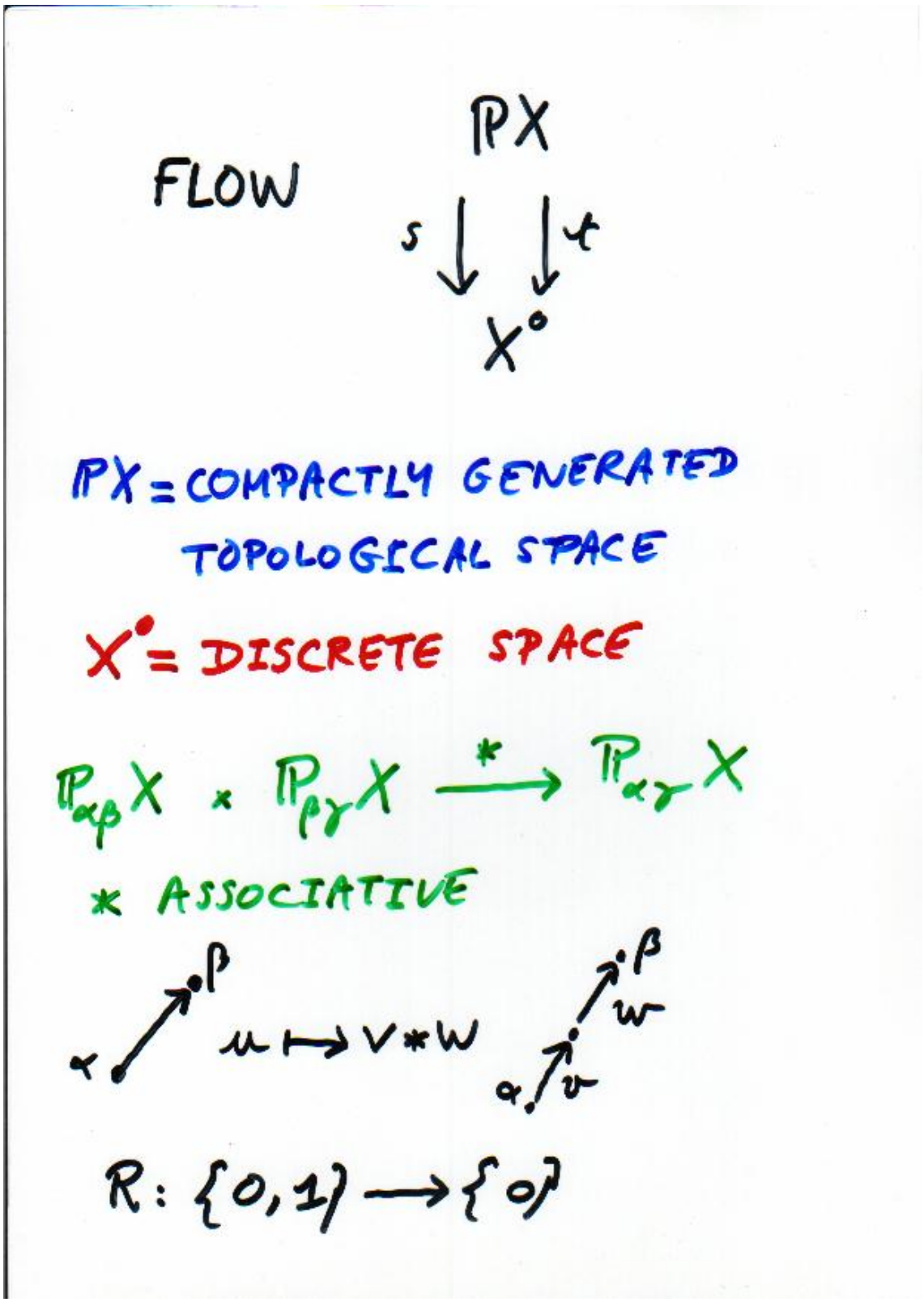
1991 *Mathematics Subject Classification.* 55P99, 68Q85.
Key words and phrases. concurrency, homotopy.



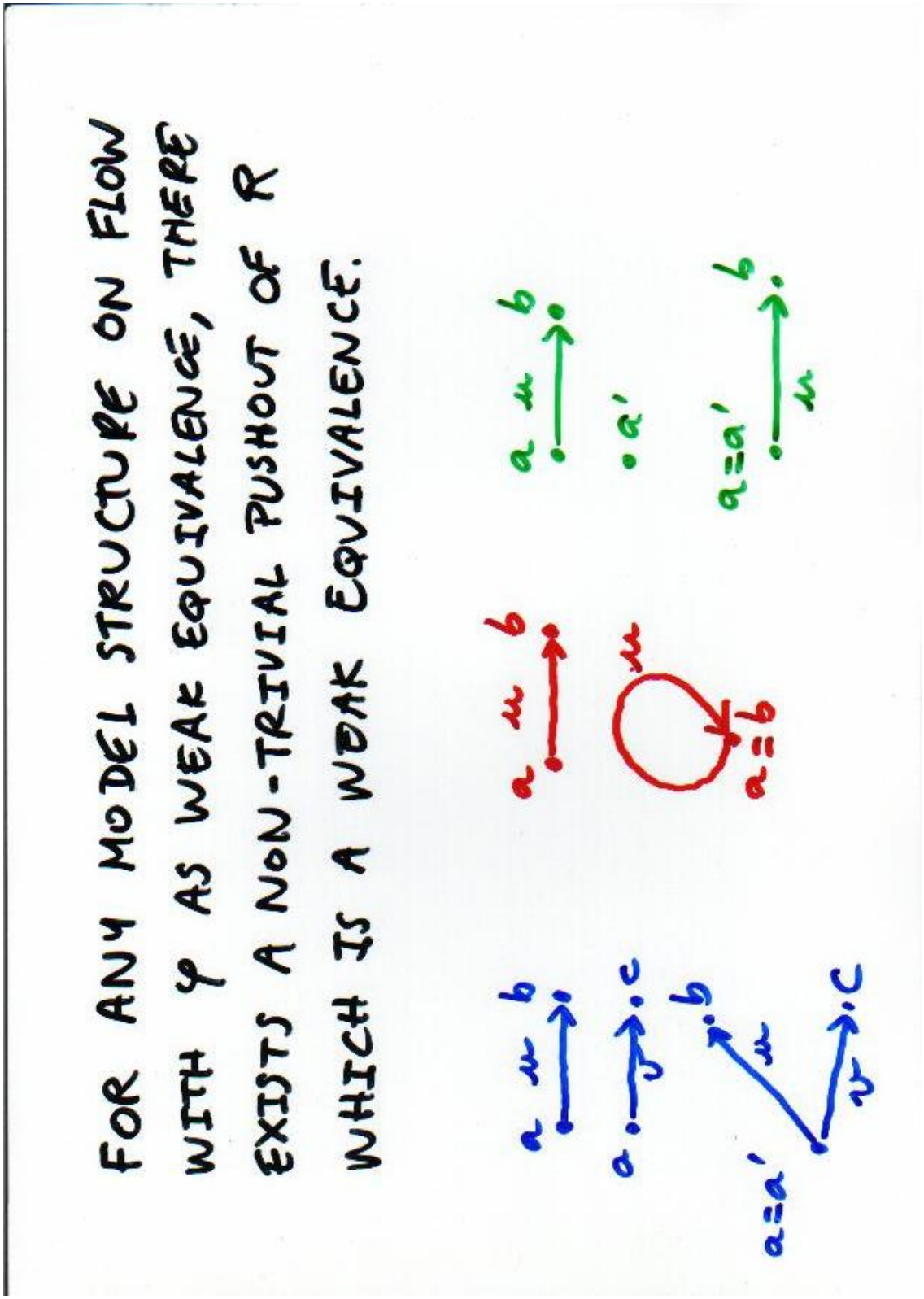
Transparent 1



Transparent 2



Transparent 3

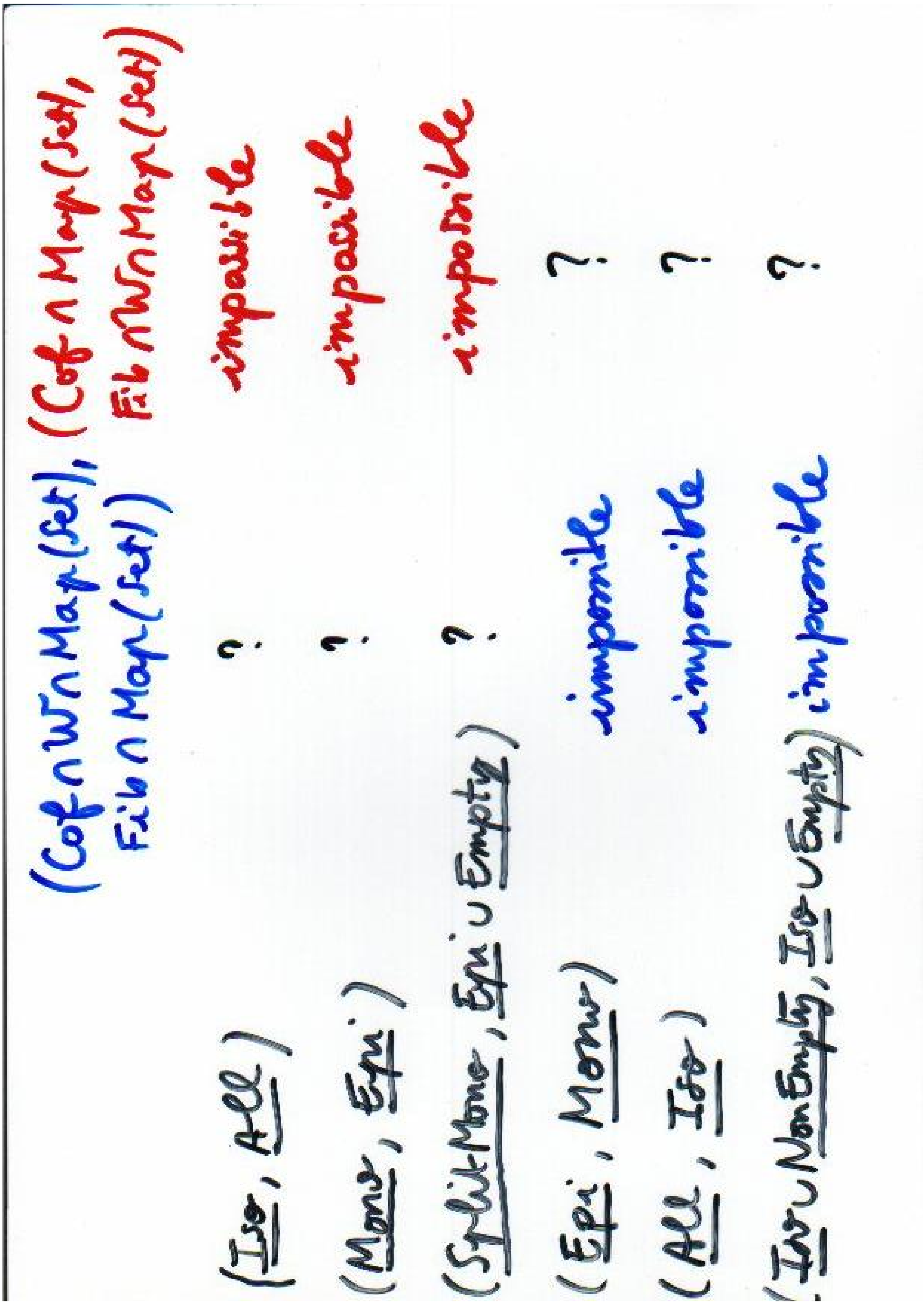


Transparent 4

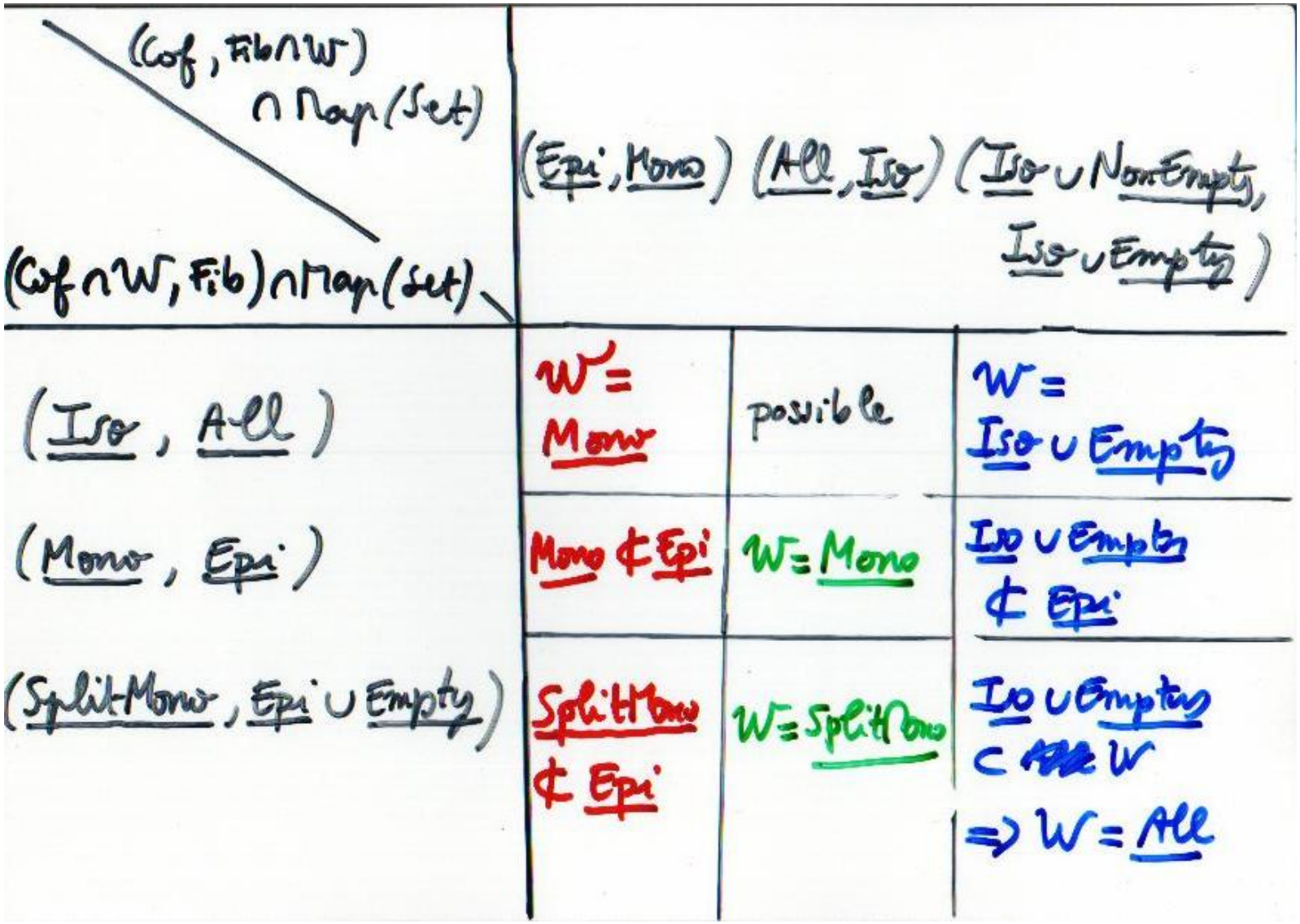
LET (Cof, Fib, W) BE A MODEL STRUCTURE SUCH THAT A WEAK EQUIVALENCE IS NEVER A NON-TRIVIAL PUSHOUT OF R. THEN

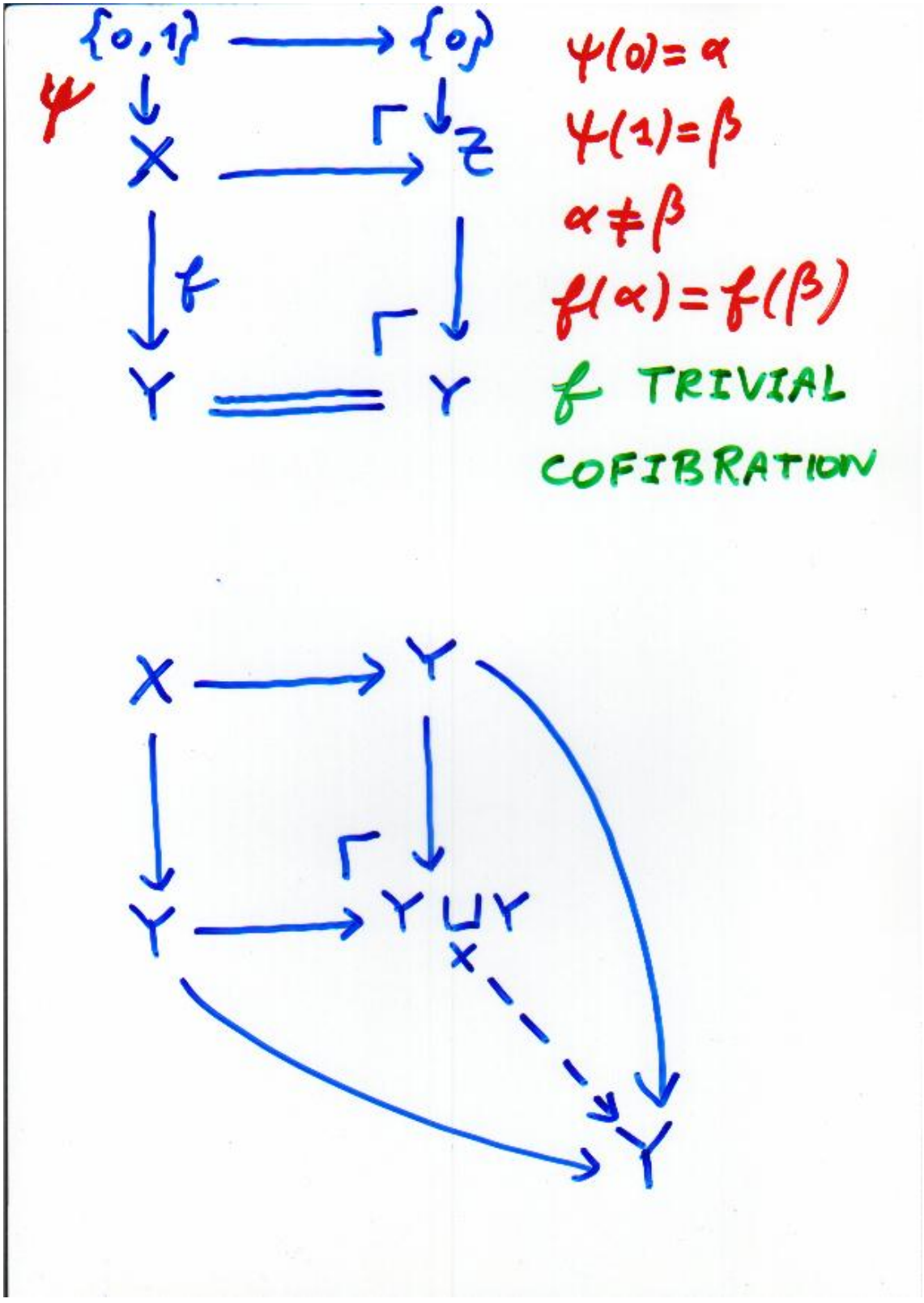
$$(Cof \cap Map(Set), Fib \cap Map(Set), W \cap Map(Set))$$

$$= (\underline{All}, \underline{All}, \underline{Iso})$$

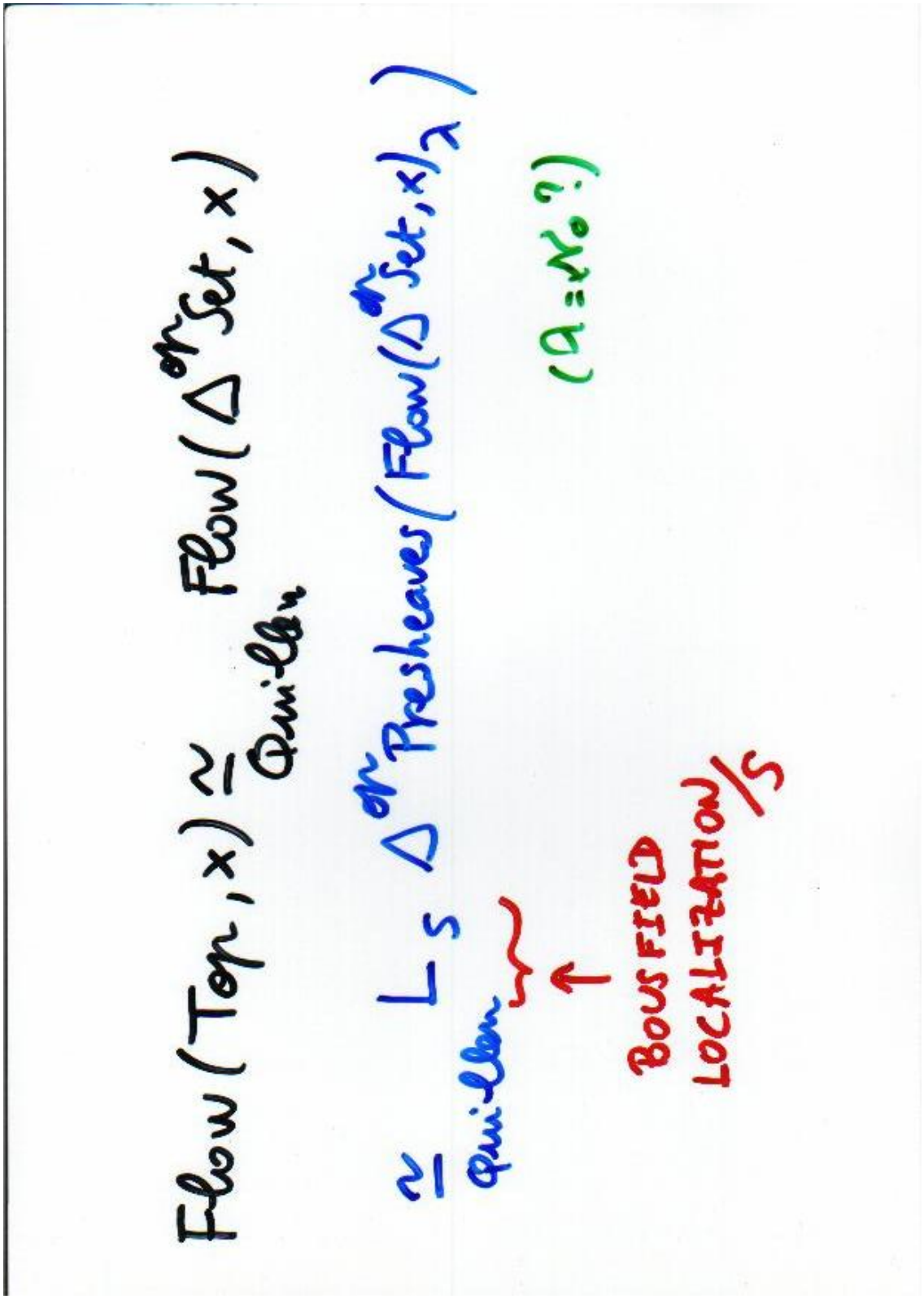


Transparent 6


(Cof, Fib ∩ W) ∩ Map(Set)
(Cof ∩ W, Fib) ∩ Map(Set)
(Epi, Mono)
(All, Iso)
(Iso ∪ NonEmpty, Iso ∪ Empty)
(Iso, All)
W = Mono
possible
W = Iso ∪ Empty
(Mono, Epi)
Mono ⊄ Epi
W = Mono
Iso ∪ Empty ⊄ Epi
(SplitMono, Epi ∪ Empty)
SplitMono ⊄ Epi
W = SplitMono
Iso ∪ Empty ⊂ W
⇒ W = All



Transparent 8



Transparent 9

References

[AHRT02] J. Adámek, H. Herrlich, J. Rosický, and W. Tholen. On a generalized small-object argument for the injective subcategory problem. *Cah. Topol. Géom. Différ. Catég.*, 43(2):83–106, 2002.

[AR94] J. Adámek and J. Rosický. *Locally presentable and accessible categories*. Cambridge University Press, Cambridge, 1994.

[BK72] A. K. Bousfield and D. M. Kan. *Homotopy limits, completions and localizations*. Springer-Verlag, Berlin, 1972. Lecture Notes in Mathematics, Vol. 304.

[Bor94a] F. Borceux. *Handbook of categorical algebra. 1*. Cambridge University Press, Cambridge, 1994. Basic category theory.

[Bor94b] F. Borceux. *Handbook of categorical algebra. 2*. Cambridge University Press, Cambridge, 1994. Categories and structures.

[Bro88] R. Brown. *Topology*. Ellis Horwood Ltd., Chichester, second edition, 1988. A geometric account of general topology, homotopy types and the fundamental groupoid.

[Dug01a] D. Dugger. Combinatorial model categories have presentations. *Adv. Math.*, 164(1):177–201, 2001.

[Dug01b] D. Dugger. Universal homotopy theories. *Adv. Math.*, 164(1):144–176, 2001.

[Dug03] D. Dugger. Notes on Delta-generated spaces, 2003.

[Gau03a] P. Gaucher. Comparing globular CW-complex and flow, 2003. arXiv:math.AT/0308063.

[Gau03b] P. Gaucher. Homotopy branching space and weak dihomotopy, 2003. arXiv:math.AT/0304112.

[Gau03c] P. Gaucher. A model category for the homotopy theory of concurrency. *Homology, Homotopy and Applications*, 5(1):p.549–599, 2003.

[Gau04] P. Gaucher. S-homotopy as an analogue of homotopy of spaces, 2004. arXiv:math.AT/0401033.

[GJ99] P. G. Goerss and J. F. Jardine. *Simplicial homotopy theory*. Birkhäuser Verlag, Basel, 1999.

[Hir03] P. S. Hirschhorn. *Model categories and their localizations*, volume 99 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2003.

[Hov99] M. Hovey. *Model categories*. American Mathematical Society, Providence, RI, 1999.

[Lew78] L. G. Lewis. *The stable category and generalized Thom spectra*. PhD thesis, University of Chicago, 1978.

[May99] J. P. May. *A concise course in algebraic topology*. University of Chicago Press, Chicago, IL, 1999.

[Qui67] D. G. Quillen. *Homotopical algebra*. Springer-Verlag, Berlin, 1967.

[Smi] J. Smith. A really convenient category of topological spaces. unpublished.

Preuves Programmes et Systèmes, Université Paris 7–Denis Diderot, Case 7014, 2 Place Jussieu, 75251 PARIS Cedex 05, France

E-mail address: gaucher@pps.jussieu.fr

URL: http://www.pps.jussieu.fr/~gaucher/