## From $\lambda$-terms to MELL Proof-Nets

## Recall that...

Two forms for atomic formulae : $p$ and $\underline{p}$.

Negation of formulae is defined as follows :

$$
\begin{array}{lclclcl}
p^{\perp} & := & \underline{p} & \quad & \underline{p}^{\perp} & := & p \\
(A \parr B)^{\perp} & := & A^{\perp} \otimes B^{\perp} & \quad & (A \otimes B)^{\perp} & := & A^{\perp} \parr B^{\perp} \\
(?A)^{\perp} & := & !A^{\perp} & \quad & (!A)^{\perp} & := & ?A^{\perp}
\end{array}
$$



## Reduction rules and equations for MELL Proof-Nets

See

`http://www.pps.jussieu.fr/~kesner/enseignement/mpri/ll/RE-MELL-proofnets1.pdf`



## Reduction relation for MELL Proof-Nets

Let us consider :

$$
\begin{array}{lcl}
E & = & A \cup B \\
CE & = & \{Ax\text{-}cut, \parr\text{-}\otimes, w\text{-}b, d\text{-}b, c\text{-}b, b\text{-}b\} \\
R & = & CE \cup \{\mathtt{U}, \mathtt{V}\}
\end{array}
$$

The reduction relation on MELL proof-nets is generated by the reduction rules $R$ and congruence axioms $E$ :

$$p \rightarrow_{R/E} p' \text{ iff } \exists p_1, p_2 \; p \sim_E p_1 \rightarrow_R p_2 \sim_E p'$$

## Termination properties of proof-nets

(Girard)

The reduction system generated by the reduction rules $CE$ is SN.

(DiCosmo-Guerrini)

The reduction system generated by the reduction rules $CE \cup \mathtt{V}$ modulo the axioms $E$ is SN.

(Polonovski)

The reduction relation $R/E$ is SN.



## From simply typed $\lambda$s-terms to MELL Proof-Nets

Translate types and type derivations as follows :

`http://www.pps.jussieu.fr/~kesner/enseignement/mpri/11/lambdas-proofnets2.pdf`



## Translating $\lambda$s-Reduction on Typed Terms

**Theorem [From $\lambda$s to MELL]** Let $\Gamma \vdash_{\lambda\mathsf{s}} t : A$ and $t \to_{\lambda\mathsf{s}} t'$, then $T(\Gamma \vdash_{\lambda\mathsf{s}} t : A) \to^*_{R/E} C[T(\Gamma' \vdash_{\lambda\mathsf{s}} t : A)]$ for some $\Gamma' \subseteq \Gamma$ and some MELL-context made only of weakenings.

- Which $\lambda$s-steps are strictly/weakly translated ?
- How SN for $\lambda$s-typed terms can be concluded from SN for MELL Prof-Nets ?



## The $\sigma$-equivalence in $\lambda$-calculus

$$
\begin{array}{llll}
(\lambda x.\lambda y.U)V & \equiv_{\sigma_1} & \lambda y.(\lambda x.U)V & \text{if } y \notin FV(V) \\
(\lambda x.UV)W & \equiv_{\sigma_2} & ((\lambda x.U)W)V & \text{if } x \notin FV(V)
\end{array}
$$

**Lemma** If $t \equiv_\sigma t'$, then $t \equiv_\beta t'$.

**Theorem [Regnier'90]** If $t \equiv_\sigma t'$, then $\eta_\beta(t) = \eta_\beta(t')$.

## The $\sigma$-equivalence in calculi with ES

$$
\begin{array}{lcll}
(\lambda y.U)[x/V] & \equiv & \lambda y.U[x/V] & \text{if } y \notin FV(V) \\
(UV)[x/W] & \equiv & U[x/W]V & \text{if } x \notin FV(V)
\end{array}
$$