Type Isomorphisms and Game Semantics

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Introduction

(Usual) game model very near from normal forms of λ-calculus

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What does it give us which is not already in the syntax ?

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Advantages on the syntax :

- it manages dynamically $\beta\eta$-reduction
- easily extensible
- easier to express geometrical ideas

Type isomorphisms

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- Definition :

$$(t, u) \text{ with } \begin{cases} t : A \to B \\ u : B \to A \end{cases} \quad \text{and} \begin{cases} t \circ u = id_B \\ u \circ t = id_A \end{cases}$$

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existence of an equationnal system $\simeq_\varepsilon$ such that

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A and B isomorphic

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Interest : nice and syntactic problem

$\Rightarrow$ tool to decide if you have have a *good* model

A semantic approach

Olivier Laurent : semantic proof

$\Longrightarrow$ flexibility, extension to other calculi ($\lambda\mu$-calculus...)

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game model

$\sigma_t : A^* \to B^*$

$\sigma_u : B^* \to A^*$

$\sigma_u ; \sigma_t = id_{B^*}$

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isomorphic arenas

A^* and B^* identical

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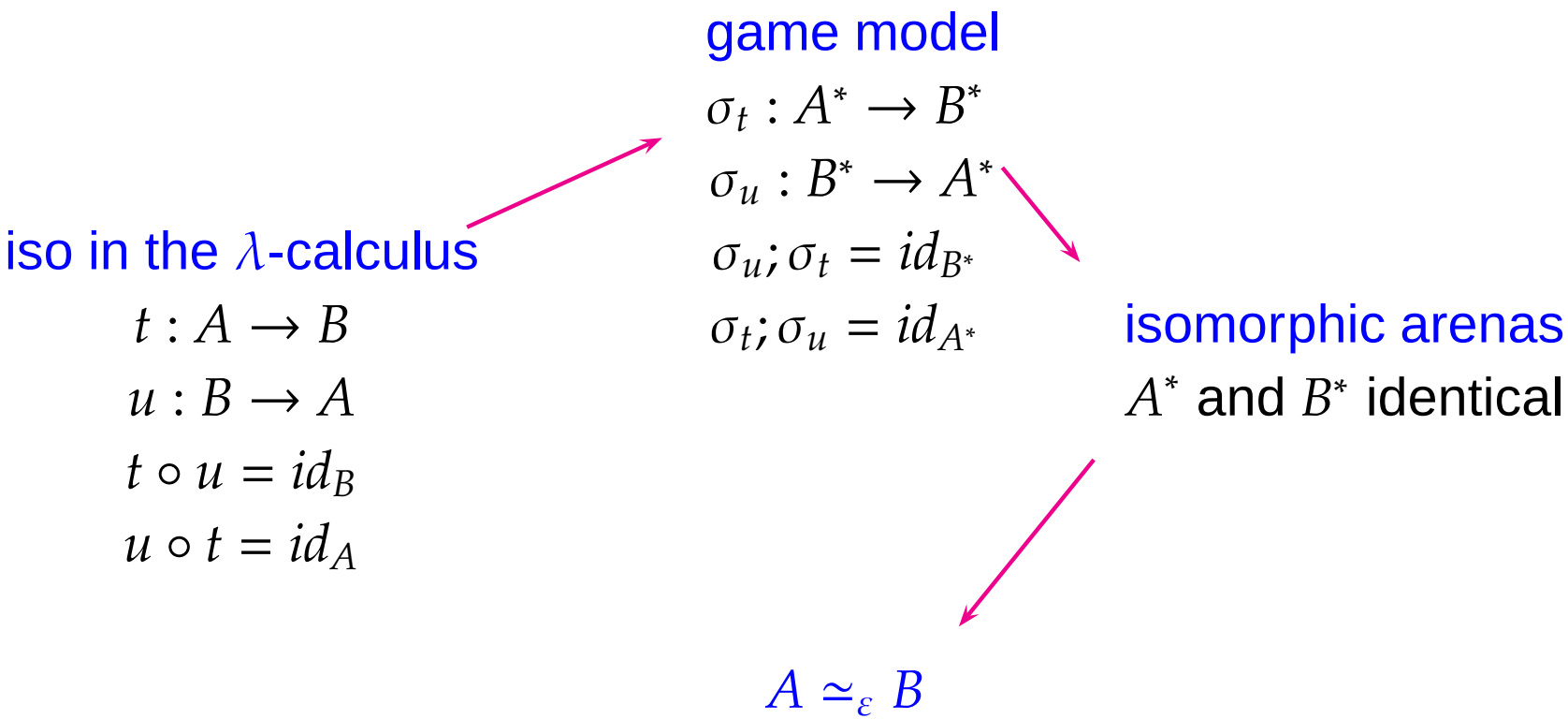

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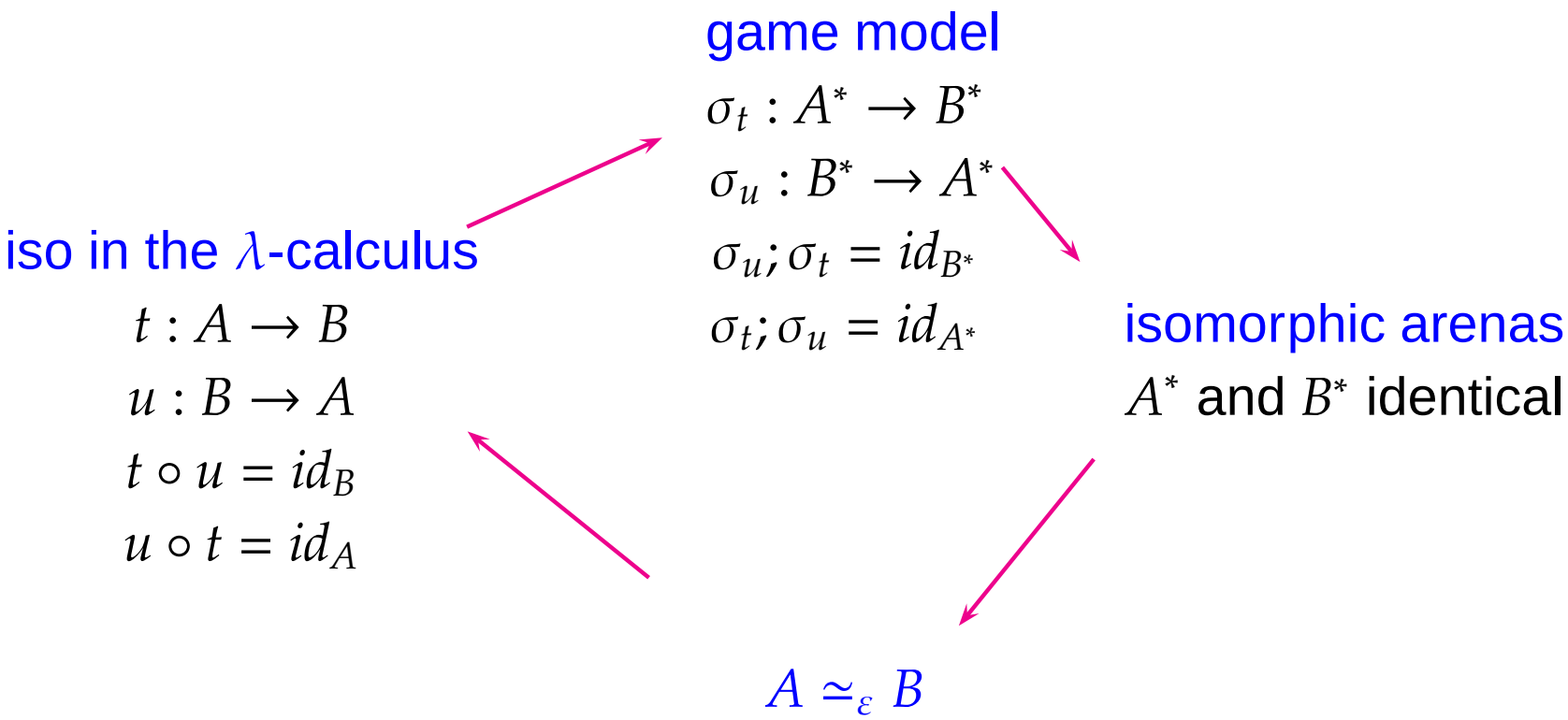

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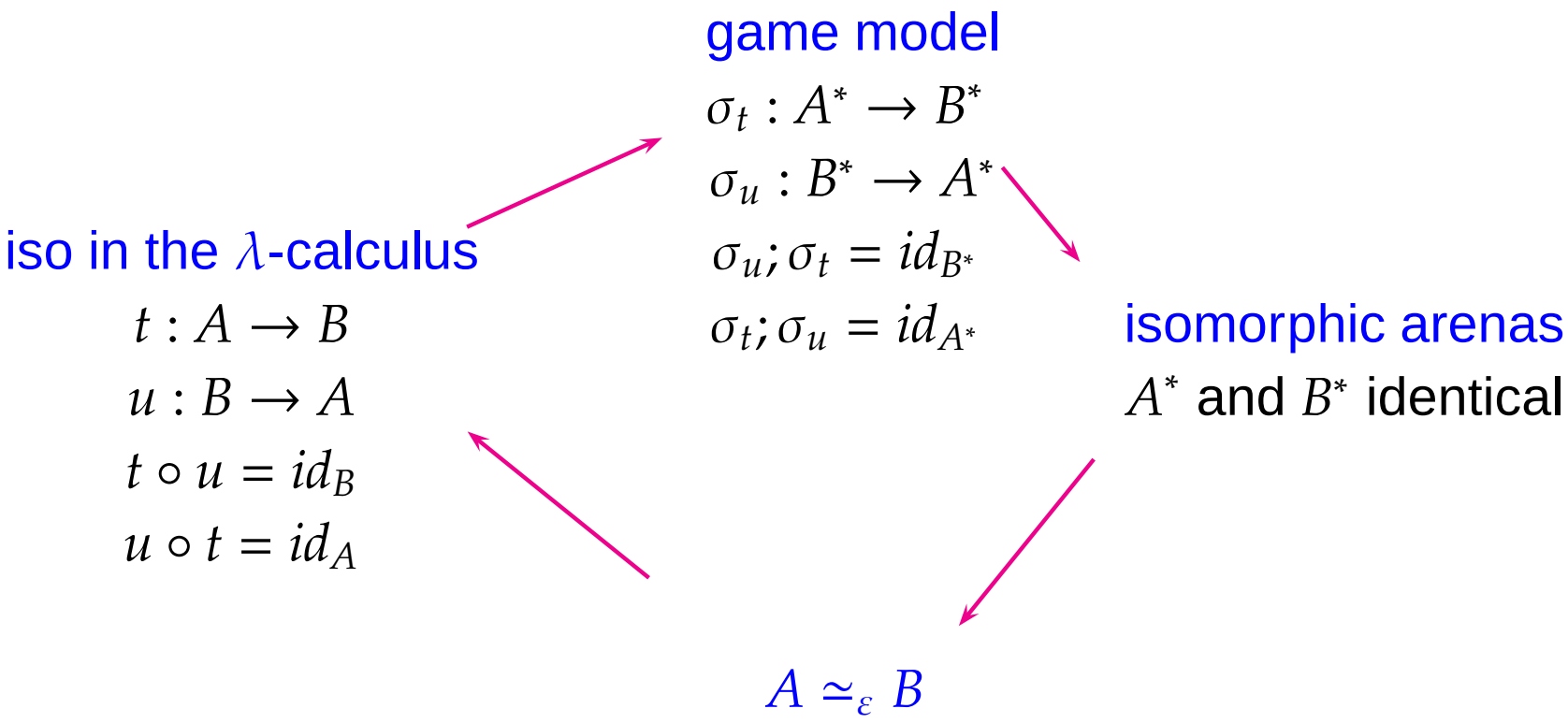

Second-order ?

System F

System F

$$\frac{}{\Gamma, x : A \vdash x : A}\ (\text{ax})$$

$$\frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x : A.t : A \to B}\ (\to I) \qquad \frac{\Gamma \vdash t : A \to B \qquad \Gamma \vdash u : A}{\Gamma \vdash tu : B}\ (\to E)$$

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$$\frac{\Gamma \vdash t : \forall X_i.A}{\Gamma \vdash t\{B\} : A[B/X_i]}\ (\forall E) \quad \frac{\Gamma \vdash t : A}{\Gamma \vdash \Lambda X_i.t : \forall X_i.A}\ (\forall I) \text{ if } X_i \notin FTV(\Gamma)$$

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$$(\lambda x.t)u = t[u/x] \qquad (\beta)$$

$$\lambda x.tx = t \quad \text{if } x \notin FT(t) \qquad (\eta)$$

$$(\Lambda X.t)\{B\} = t[B/X] \qquad (\beta 2)$$

$$\Lambda X.t\{X\} = t \quad \text{if } X \notin FTV(t) \qquad (\eta 2)$$

Game models of system F

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   complete game model for system F

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4. A. Murawski - L. Ong :

   evolving (AJM-)games for affine polymorphism

   Evolving games and essential nets for affine polymorphism (TLCA, 2001)

Polymorphic arenas

Types $\mapsto$ forests with an additionnal structure

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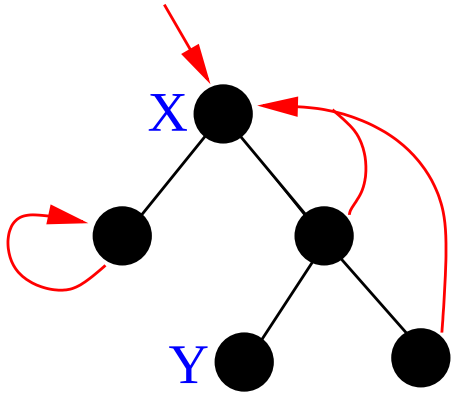
Example :



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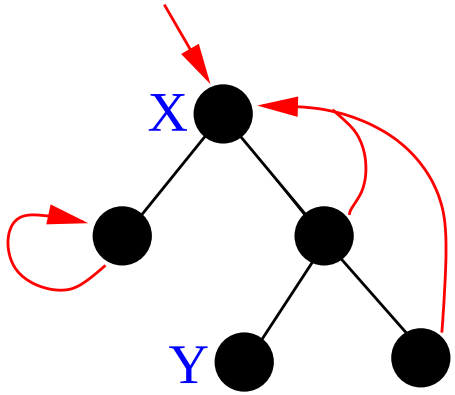

$$\forall Z.\forall U.(U \to Y \to U) \to (\forall V.V) \to X$$

Operations on arenas

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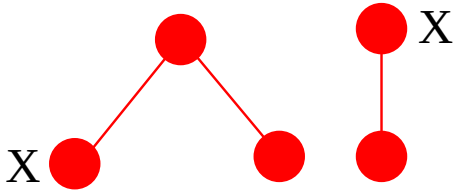
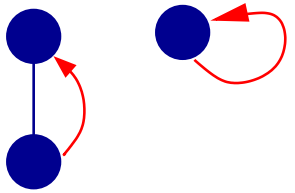
$A^* =$ 

$B^* =$ 

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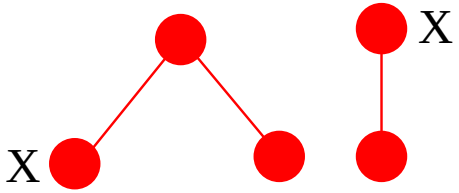
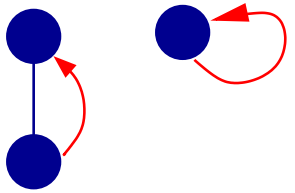
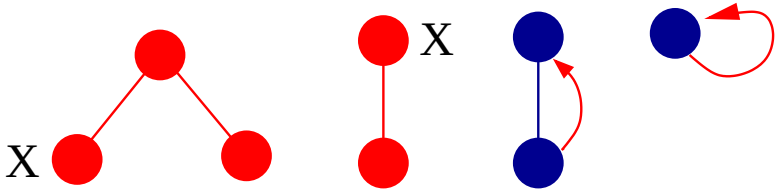
$A^* =$ 

$B^* =$ 

$(A \times B)^* =$ 

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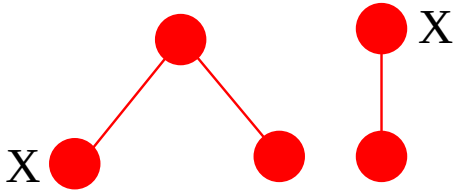
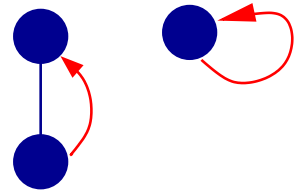
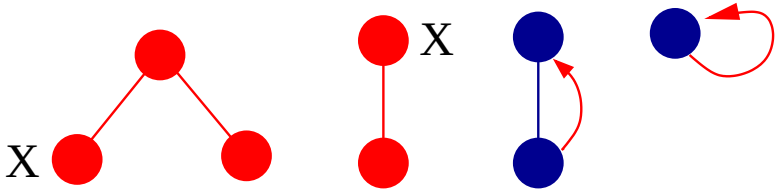
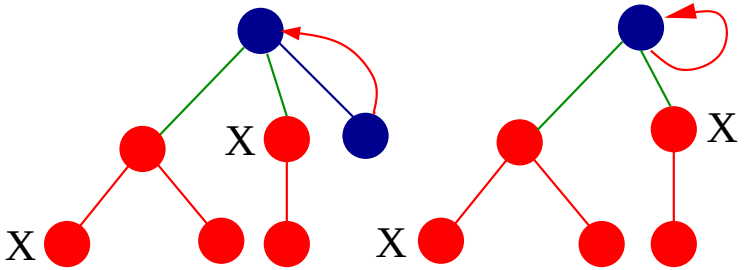
$A^* =$ 

$B^* =$ 

$(A \times B)^* =$ 

$(A \to B)^* =$ 

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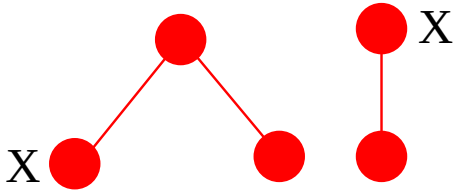
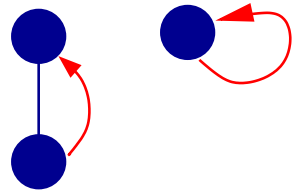
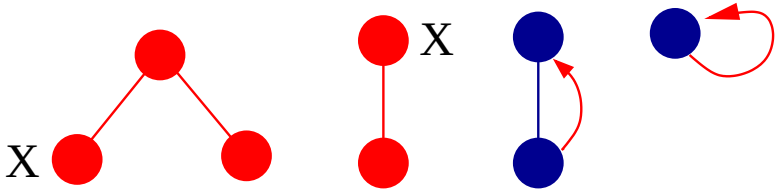
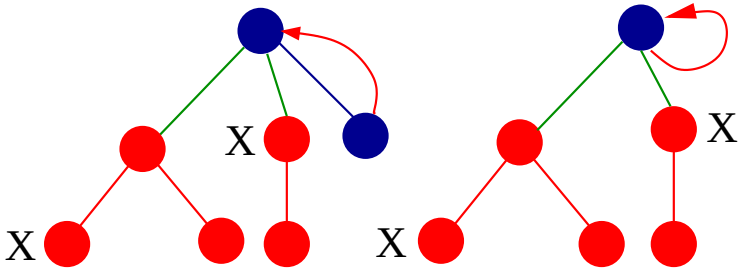
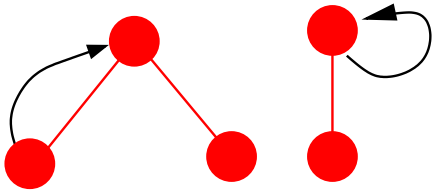
$A^* =$ 

$B^* =$ 

$(A \times B)^* =$ 

$(A \to B)^* =$ 

$(\forall X.A)^* =$ 

Play in a hyperforest

$$t = \Lambda Z.\Lambda X.\lambda x^{X}.\lambda y^{\forall Y.Y}.(y\{X \to \bot\})x : \forall Z \forall X.X \to (\forall Y.Y) \to \bot$$

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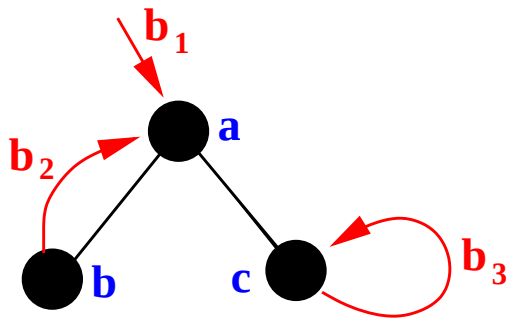

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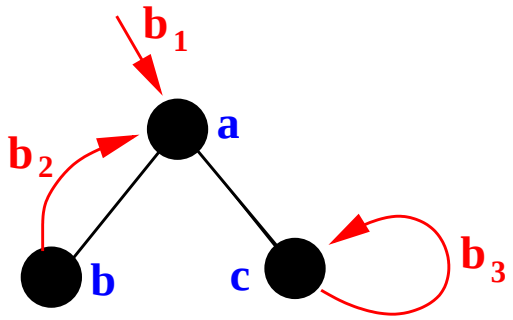

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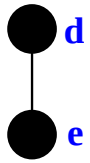
$H =$ 

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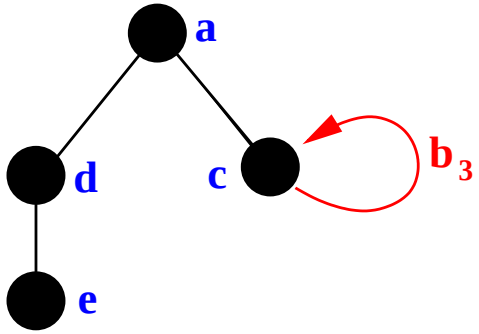

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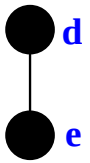
$H =$ 

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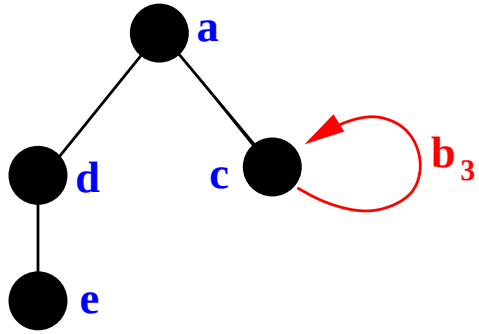

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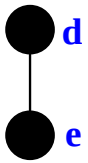
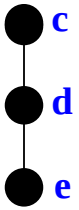
$H =$ 

$H' =$ 

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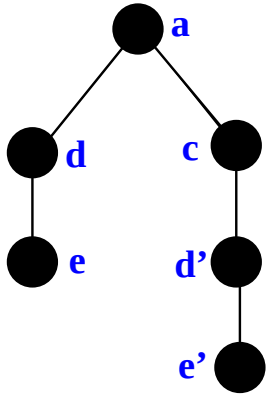

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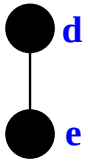
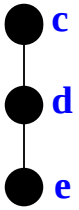
$H =$ 

$H' =$ 

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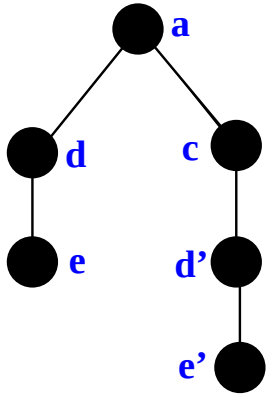

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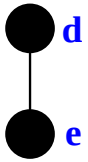
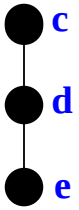
$$[d']$$

$H =$ 

$H' =$ 

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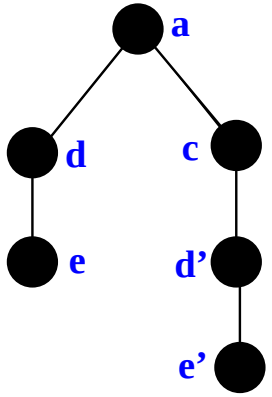

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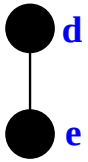
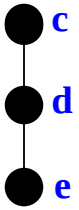
$H =$ 

$H' =$ 

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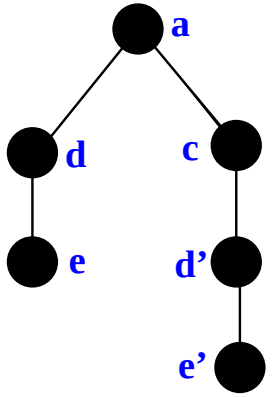

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$$[d']$$

$$[d]$$

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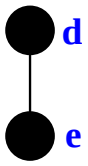
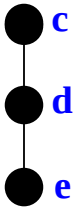
$H =$ 

$H' =$ 

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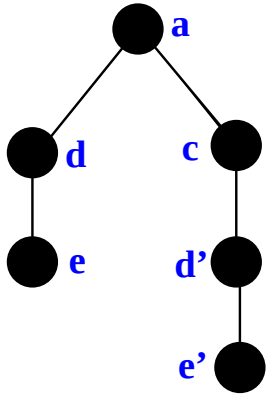

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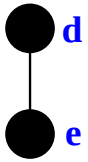

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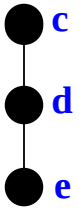

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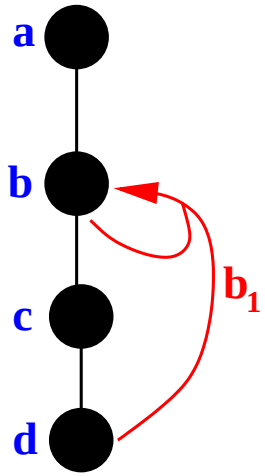

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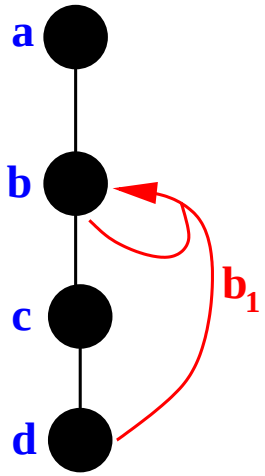

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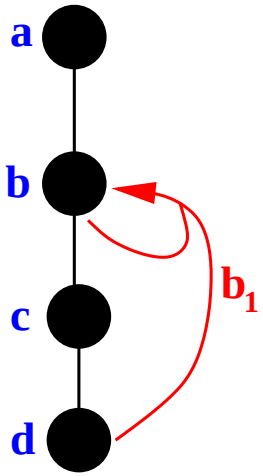

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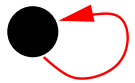
$H =$ 

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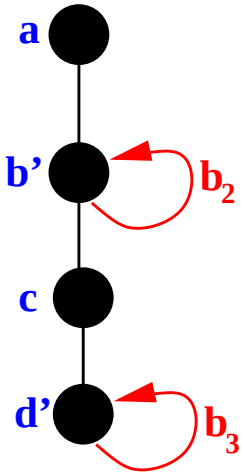

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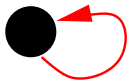
$H =$ 

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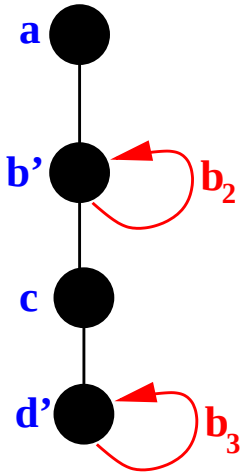

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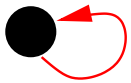
$H =$ 

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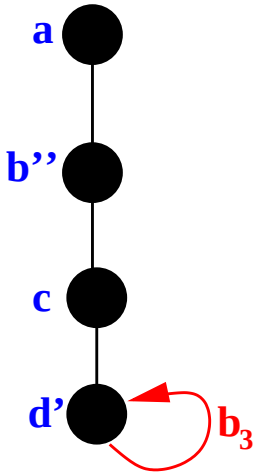

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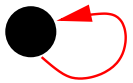
$H =$ 

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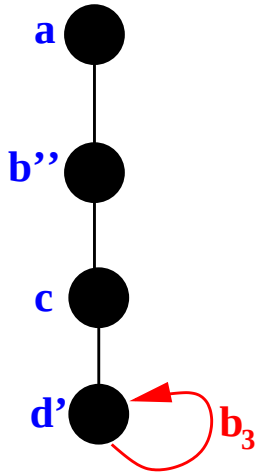
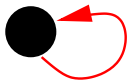


$[a]$ $\quad H =$  $\quad H' =$ 

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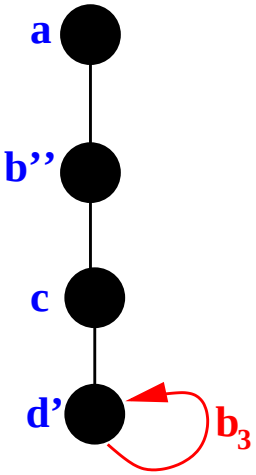
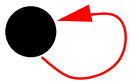


$[a]$ $\quad H =$  $\quad H' =$ 

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$$m = [c_1 : H_1^1/b_1^1; \ldots; H_{k_1}^1/b_{k_1}^1] \ldots [c_n : H_1^n/b_1^n; \ldots; H_{k_n}^n/b_{k_n}^n]$$

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Categorical models of system F :

hyperdoctrines

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THE Theorem

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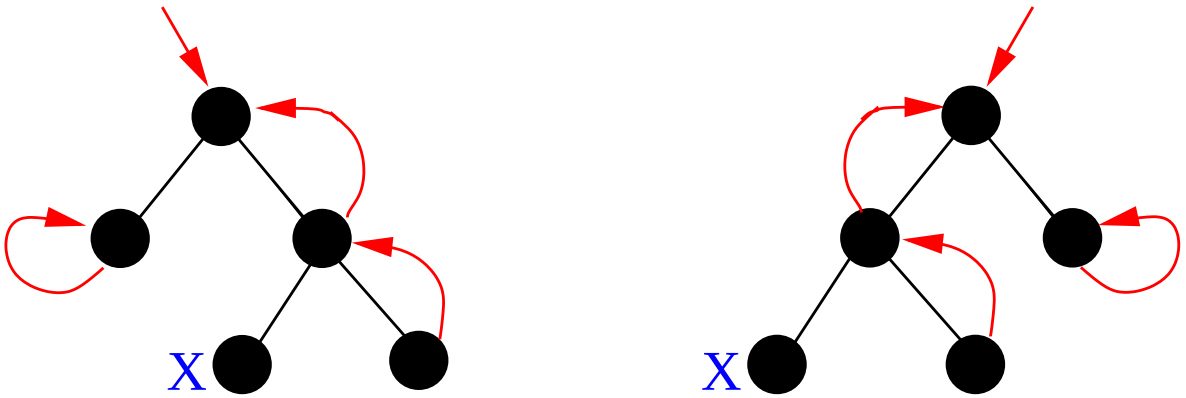
A^* and B^* *identical*



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game model

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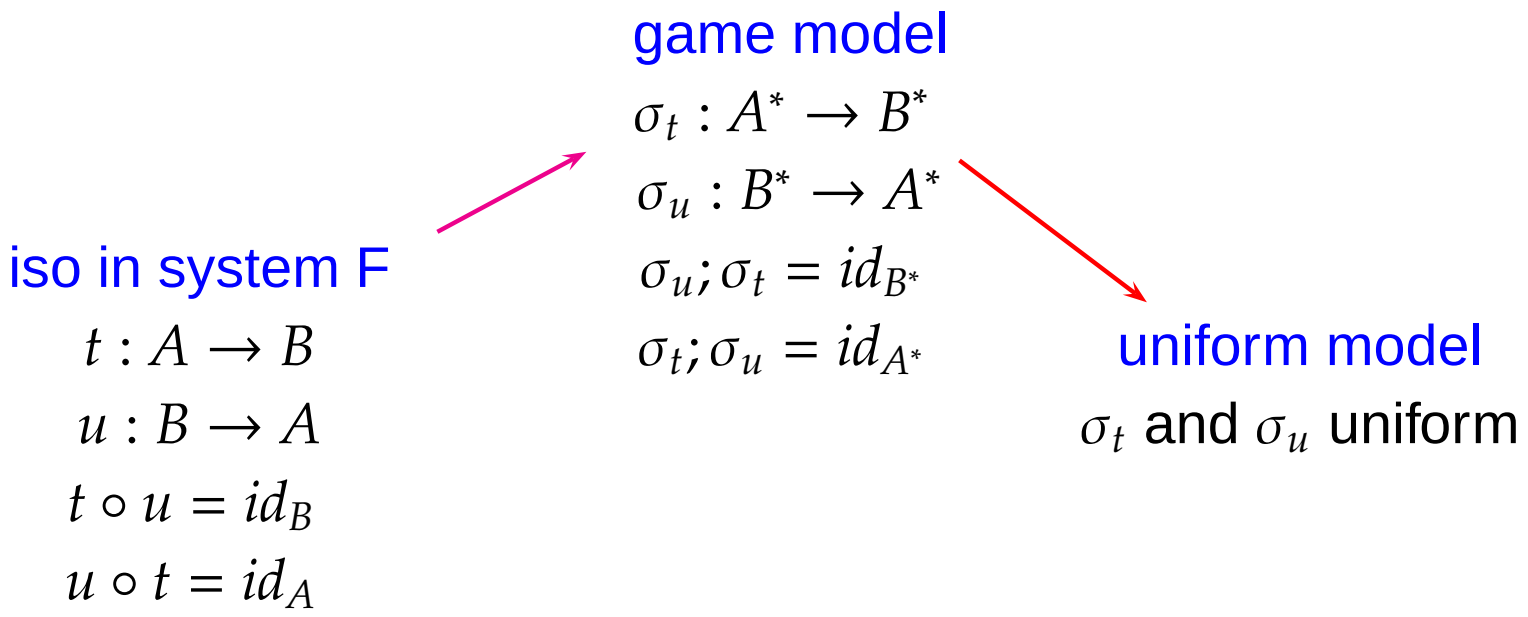

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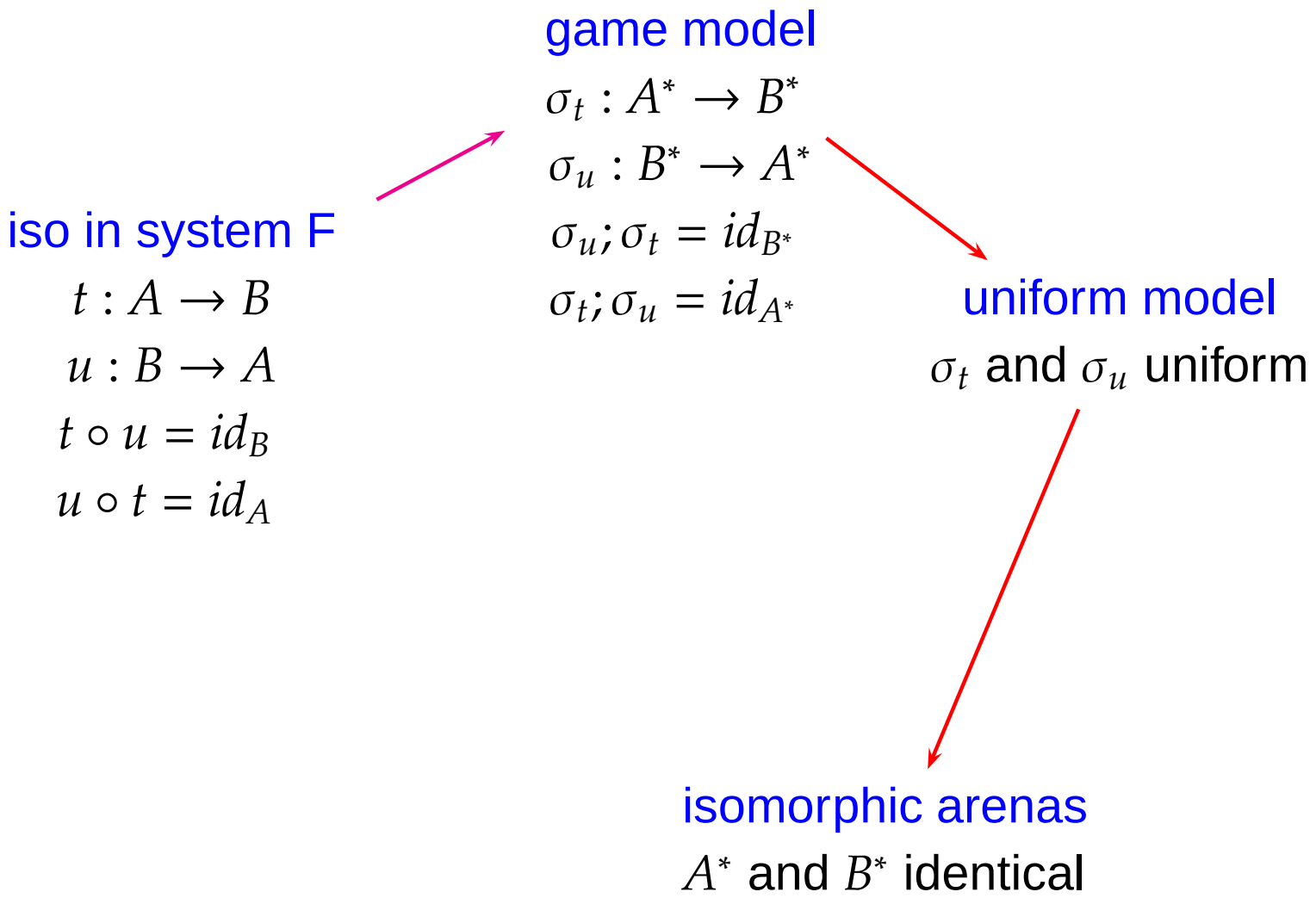

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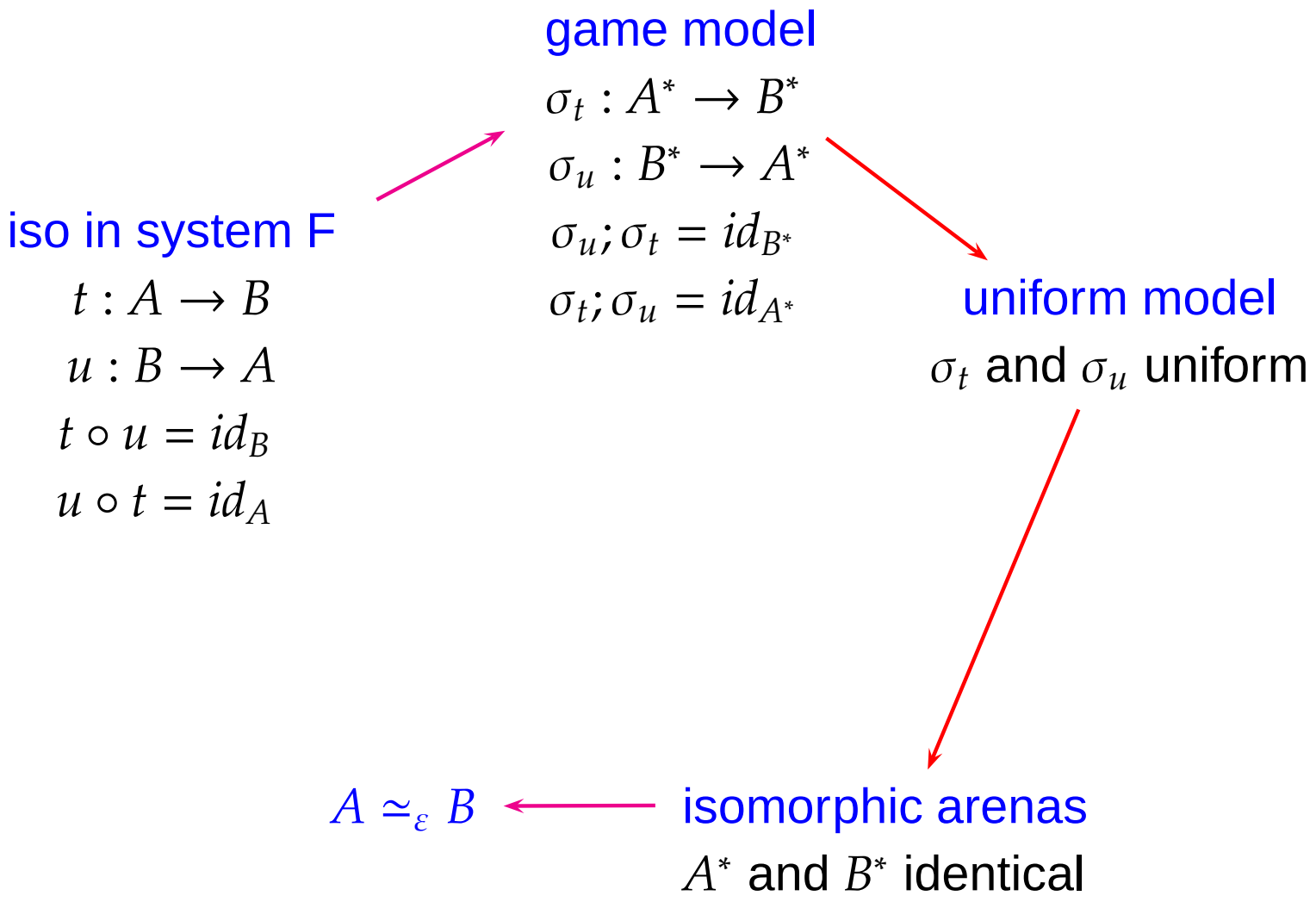

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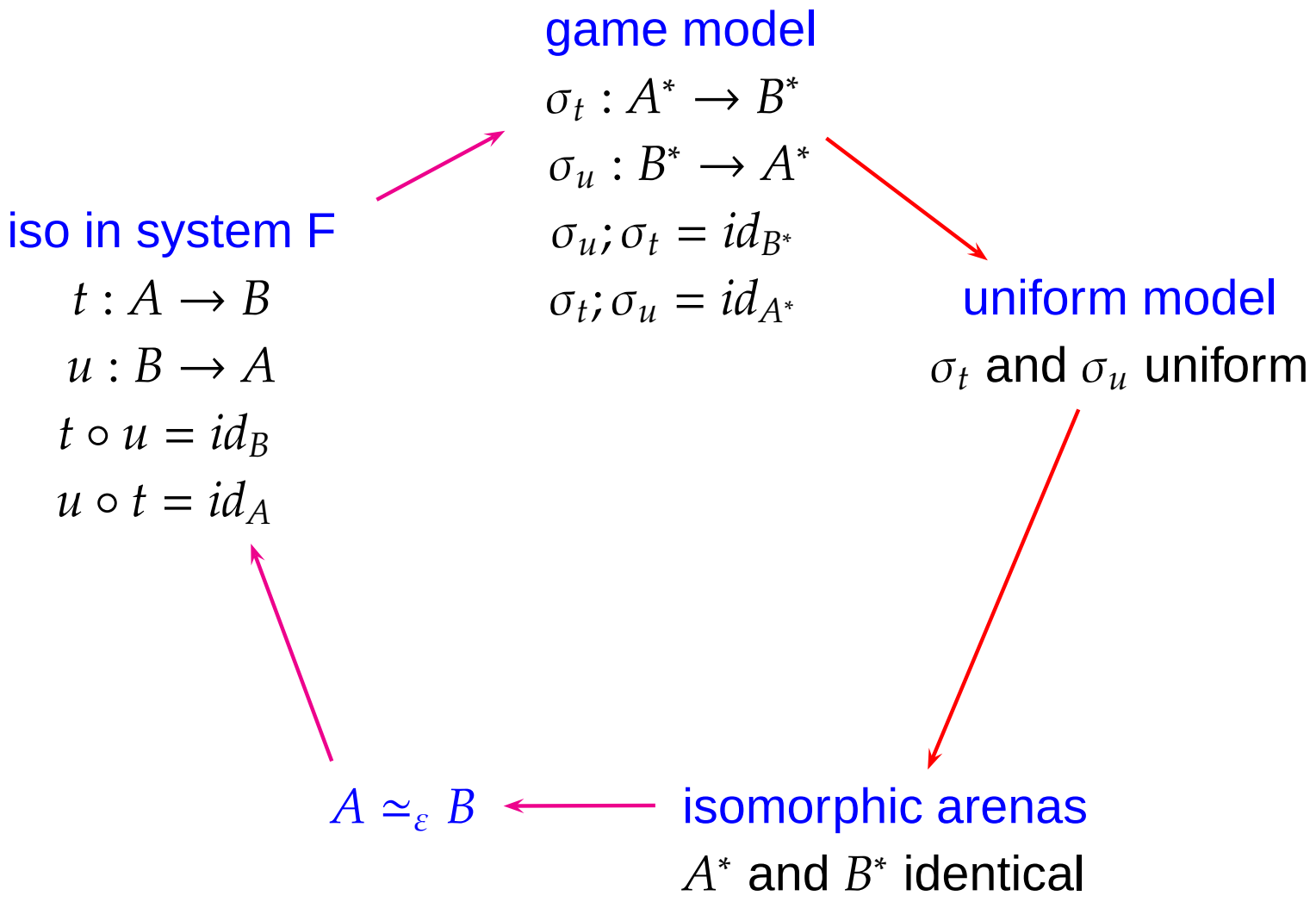

Type isomorphisms in system F

Roberto Di Cosmo :

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same isomorphisms as in λ-calculus

+

$$\forall X.\forall Y.A \simeq_\varepsilon \forall Y.\forall X.A$$

$$\forall X.A \simeq_\varepsilon \forall Y.A[Y/X] \qquad \text{if } Y \notin FTV(A)$$

$$A \to \forall X.B \simeq_\varepsilon \forall X.(A \to B) \qquad \text{if } X \notin FTV(A)$$

$$\forall X.(A \times B) \simeq_\varepsilon \forall X.A \times \forall X.B$$

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Models for second-order $\lambda\mu$-calculus :
control hyperdoctrines

$\lambda\mu$-calculus with a disjunction type

$$\frac{\Gamma \vdash t : A \mid \alpha : A, \beta : B, \Delta}{\Gamma \vdash [\alpha, \beta]t : \bot \mid \alpha : A, \beta : B, \Delta}$$ (double naming)

$$\frac{\Gamma \vdash t : \bot \mid \alpha : A, \beta : B, \Delta}{\Gamma \vdash \mu(\alpha, \beta).t : A \parr B \mid \Delta}$$ (double name abstraction)

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ISOMORPHISMS :

$$A \parr B \simeq_\varepsilon B \parr A$$

$$A \parr (B \parr C) \simeq_\varepsilon (A \parr B) \parr C$$

$$(A \to B) \parr C \simeq_\varepsilon A \to (B \parr C)$$

$$A \parr (B \times C) \simeq_\varepsilon (A \parr B) \times (A \parr C)$$

$$\forall X.(A \parr B) \simeq_\varepsilon (\forall X.A) \parr B \qquad \text{if } X \notin FTV(B)$$

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Other extensions

- Idealized Algol

  (visibility required, not innocence)

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  (totality not required)

Curry-style system F

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$$\forall X.A \simeq_\varepsilon A[\forall Y.Y/X]$$

if any X in A occurs in an Opponent position

Further directions

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  (which representation of types ?)

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- Signification of uniformity

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- Retractions